Air Quality Prediction of PM$_{10}$ through an Analytical Dispersion Model for Delhi

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ABSTRACT

An analytical dispersion model for point, line and area sources is formulated in this work. The analytical solution of an advection-diffusion equation with the Neumann (total reflection) boundary conditions for a bounded domain is obtained for point sources using the separation of variable and wind speed as a power law profile of vertical height above the ground. The downwind and vertical eddy diffusivities are considered as an explicit function of downwind distance and vertical height. The formulations for line and area sources are obtained by integrating the point source formulation in crosswind, crosswind and downwind directions, respectively. A gridded emissions inventory of Delhi City for the year 2008–09 has been developed to estimate the strength of emissions from various sources, namely vehicles, industries, power plants and domestic sources, and this has been made using primary and secondary data of PM$_{10}$ (particulate matter with aerodynamic diameter ≤ 10 µm). Dispersion models generally require steady and horizontally homogeneous hourly surface and upper air meteorological observations. However, hourly meteorological observations are not easily available for most of the locations in India. To overcome this limitation, meteorological variables are computed using the Weather Research and Forecast (WRF) model (version 3.1.1) developed by the National Center for Atmospheric Research (NCAR). The performance of this analytical model is evaluated through concentrations of PM$_{10}$ monitored at different locations in Delhi. The observed data for the period December 2008 has been obtained from the Central Pollution Control Board (CPCB). The model’s predicted values are found to be in good agreement with the observed values. However, the model results depend on the accuracy of source strength data, i.e., the estimated values of emission rates for the various air pollutants.

Keywords: Analytical model; PM$_{10}$; Weather Research and Forecast (WRF); Emission inventory.

INTRODUCTION

The atmospheric diffusion equation (Seinfeld, 1986) has long been used to describe the dispersion of airborne pollutants in a turbulent atmosphere. The use of analytical solutions of this equation was the first and remains the convenient way for modelling the air pollution study (Demuth, 1978). Air dispersion models based on its analytical solutions posses several advantages over numerical models, because all the influencing parameters are explicitly expressed in a mathematically closed form. Analytical models are also useful in examining the accuracy and performance of numerical models. In practice, most of the estimates of dispersion are based on the Gaussian plume model, which assumes the constant wind speed and turbulent eddies with height. Hinrichsen (1986) compared a non-Gaussian model, in which wind speed and turbulence, are not constant with height and non Gaussian model agreed better with the observed data.

Several efforts have also been made for non-Gaussian models of point and line sources. Since observational studies show that the wind speed and eddy diffusivity vary with vertical height above the ground (Stull, 1988). Analytical solutions of the advection diffusion equation, with wind speed and vertical eddy diffusivity both as function of vertical height, are well known for point and line sources bounded by Atmospheric Boundary Layer (ABL) (Seinfeld, 1986; Lin and Hildemann, 1996). Taylor’s (1921) analysis and statistical theory suggest that the eddy diffusivity depends on the downwind distance from the source (Arya, 1995). The advection diffusion equation has also solved analytically with wind speed as function of height and eddy diffusivity as a function of downwind distance from the source (Sharan and Modani, 2006). Thus in general, the eddy diffusivity should be a function of both vertical above the ground and downwind distance from the source (Mooney and Wilson, 1993). Recently, Sharan and Kumar (2009) formulate the advection diffusion equation considering the wind speed as a function of vertical height and vertical eddy diffusivity as a function of both vertical height and downwind distance from the source applicable only for point source release in reflecting boundary condition. However,
downwind eddy diffusivity has not been considered, which is important in low wind condition.

In addition to these, the few studies have been made for analytical solution of the advection diffusion equation for area sources. Although, Park and Baik (2008) have solved the advection diffusion equation analytically for finite area source with wind speed and vertical eddy diffusivity as power function of vertical height in unbounded region, which is not applicable practically, since most of the industrial stacks, vehicular sources and domestic sources are located within boundary as dispersion of air pollutants, emitted near the surface, are largely diluted and confined within it. In that study, vertical eddy diffusivity has been parameterized only as a power law function of vertical height and that model has examined only for special test cases, not validated with observed data.

The dispersion of a pollutant is completely governed by the local meteorological variables and therefore this information is a crucial input for dispersion modelling. Analytical model requires hourly surface and upper air meteorological observations for simulating the pollutant dispersion. However, meteorological observations with such frequency are not available for most of the locations in India. To overcome this difficulty, the hourly meteorological variables are obtained from prognostic high-resolution simulations of WRF model. An offline coupling of WRF and AERMOD model has been used to simulate the concentration of PM$_{10}$ dispersion over Pune city by Kesarkar et al. (2007). In that study, sensitivity of WRF model with respect to planetary boundary layer (PBL) and land surface model (LSM) has not been examined. In addition to this, AERMOD is a Gaussian model, which suffers from few limitations like-these are applicable only for homogenous wind fields and not applicable in low wind conditions.

To overcome these limitations, the objective of the present study is to develop analytical models for dispersion of air pollutants released from point, line and area sources by considering total reflection at ground and top of ABL (Neumann boundary conditions). The analytical solution of advection diffusion equation for above described boundary condition is derived using the separation of variable technique with wind speed as a power law function of vertical height above the ground. The downwind and vertical eddy diffusivities are considered as an explicit function of vertical height and downwind distance from the source, which is also applicable to simulate the concentration in low wind condition. Hourly meteorological variables have been simulated using WRF model. Sensitive analysis of WRF model with respect to PBL and LSM has been examined to find out the best physical parameterization option. The newly developed analytical dispersion model for point, line and area sources is evaluated against observed data of PM$_{10}$ at various locations of Delhi, obtained from CPCB.

AIR QUALITY MODELING STUDY FOR DELHI CITY

Delhi (Latitude 28°35’N, Longitude 77°12’E) is situated in the northern part of India. The river Yamuna forms the eastern boundary of the city. It is situated between the Great Indian Desert (Thar Desert) of Rajasthan to the west, the central hot plains to the south and the cooler hilly region to the north and east. It has a semi-arid climate with high variation between summer and winter temperatures. Because of Delhi’s proximity to the Himalayas, cold waves from the Himalayan region dip temperatures across it. The average annual rainfall is approximately 714 mm, most of which falls in the Monsoon season (Economy survey of Delhi, 2008–09). Due to the worst meteorological scenario, the most important season in Delhi is winter, which starts in December and ends in February. This period is dominated by cold, dry air and ground-based inversion with low wind conditions, which occur frequently and increases the concentration of pollutants (Anfossi et al. (1990)). The summer (March, April and May) is governed by high temperature and high winds, while the monsoon (June, July, August) and post-monsoon (September, October, November) have moderate temperature and moderate wind conditions. Delhi, the capital city of India with 16.9 million inhabitants in 2007–08 spread over 1483 km$^2$. Due to the presence of large number of industries and migration of people from neighboring states, nearly 5.63 million vehicles were plying on Delhi roads in 2007–08 (Economy survey of Delhi, 2008–09). Delhi has one of the highest road densities in India as 1749 km of road length per 100 km$^2$. Its high population growth with high economic growth has resulted in ever-increasing demand for transportation and has created excessive pressure on the city’s existent transport infrastructure. Like many other cities in the developing world, it faces acute transport management problems leading to air pollution, congestion and resultant loss of productivity.

DESCRIPTION OF THE MODELLING SYSTEM

Analytical Dispersion Model

The steady state advection-diffusion equation for dispersion of a non reactive contaminate released from continuous source is described as:

$$
\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( K_x(x,z) \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y(x,z) \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z(x,z) \frac{\partial C}{\partial z} \right),
$$

(1)

where $x$, $y$, and $z$ are coordinates in the along-wind, crosswind, and vertical directions, respectively. $C$ is the mean concentration of pollutants, and $U(z)$ is the mean wind speed in downwind direction. $K_x(x,z)$, $K_y(x,z)$ and $K_z(x,z)$ are eddy diffusivities of pollutants in the along wind, crosswind and vertical directions respectively.

(i) Eq. (1) is subject to the Neumann Boundary Condition (total reflection) conditions, in which, $h$ is the top of the inversion/mixed layer:

$$
-K_x(x,z) \frac{\partial C(x,y,z)}{\partial z} = 0 \quad \text{at} \quad z = 0
$$

(2)

$$
-K_x(x,z) \frac{\partial C(x,y,z)}{\partial z} = 0 \quad \text{at} \quad z = h
$$

(3)
respectively at height \( z = z_r \).

The height dependent wind speed can be expressed as

\[
U(z) \cdot C(0, y, z) = Q_p
\]

where \( \delta \) is the Dirac delta function.

The transport of contaminant emitted from a source primarily depends on the wind speed \( U \). The formulations of the commonly used dispersion models in air quality studies assume wind speed to be constant. However, it is well known that wind speed increases with height in the lower part of the atmospheric boundary layer (Arya, 1999). The height dependent wind speed can be expressed as

\[
U(z) = a z^p, \quad a = U(z_r)z_r^{-p}
\]

where \( U(z_r) \) is the wind speed at reference height \( z_r \) and \( p \) depends on atmospheric stability.

In dispersion models, \( K_z \) is parameterized as a function of the height \( z \) above the ground only (Lin and Hildemann, 1996; Park and Baik, 2008). However, based on the Taylor’s analysis and statistical theory, it is revealed that the eddy diffusivity depends on the downwind distance \( x \) and \( z \) (Mooney and Wilson, 1993; Sharan and Kumar, 2009). The modified form of \( K_z(x, z) \) and \( K_x(x, z) \) are given as:

\[
K_z(x, z) = K'_z(z)g(x), \quad K_x(x, z) = K'_x(z)g(x),
\]

where \( K'_z(z) \) and \( K'_x(z) \) are the values of eddy diffusivity at height \( z = z_r \).

The analytical solution of Eq. (1) for the profiles of wind speed (Eq. (7)) and eddy diffusivity (Eq. (8)–Eq. (12)), with boundary conditions Eq. (2)–Eq. (6) is obtained as in Appendix A:

\[
C(x, y, z) = \frac{Q_p}{\sqrt{2\pi} \sigma_y} \left[ \frac{p+1}{ah^{p+1}} \right] \exp \left\{ \frac{2}{ah^{p+1}} \left[ (y - y_s)^2 \right] \right\}
\]

Line Source Model is Extended from Point Source Solution

A line source can be considered as a superposition of point sources. The solution for finite line source can be obtained by integrating point source solution from \( y_s = y_1 \) to \( y_2 \) with unit source strength \( Q_p \).

\[
C(x, y, z) = \frac{Q_p}{\sqrt{2\pi} \sigma_y} \left[ \frac{p+1}{ah^{p+1}} \right] \exp \left\{ \frac{2}{ah^{p+1}} \left[ (y - y_s)^2 \right] \right\}
\]

where \( \sigma_y \) is the standard deviation of concentration distribution in the crosswind direction.

The analytical solution of Eq. (1) for the profiles of wind speed (Eq. (7)) and eddy diffusivity (Eq. (8)–Eq. (12)),
uniform strength $Q_a$ per unit area, the solution is obtained by integrating finite line source solution from $x_s = x_1$ to $x_2$.

$$C(x, y, z) = \int_{x_1}^{x_2} C(x - x_s, y, z) dx_s$$

(16)

The R.H.S. term $C(x - x_s, y, z)$ in Eq. (16) is equivalent to the finite line source as obtained in above section. The source strength $Q_s$ is replaced by $Q_a$ in area source model.

Eq. (16) is the solutions for finite area sources and can be solved using numerical integration. The concentration from this analytical model can be estimated by using the emission inventory and meteorological data as input parameters to the models.

**Methodology to Estimate the Emission Rate of PM$_{10}$ in Study Area**

First of all, a gridded emission inventory of PM$_{10}$ has been developed over an area of 26 km $\times$ 30 km of Delhi and the total area has been divided into 195 square grids of size 2 km $\times$ 2 km. Emission of PM$_{10}$ has been estimated in each grid due to all anthropogenic sources viz., domestic, industries, power plants and vehicles using the primary and secondary data of the year 2008–09. The emissions of PM$_{10}$ from each category of vehicle, in each grid, are estimated using the number of vehicles (monitored by Central Road Research Institute (CRRI)), emission factors of respective pollutants (estimated by ARAI, 2007) and road length in that grid using the International Vehicular Emission (IVE) model (IVEM, 2008), which is developed by United States Environmental Protection Agency (USEPA). The diurnal variation of different type of vehicles is taken from the study by Padma et al. (2011). The emission from domestic sources has been estimated on the basis of fuel consumption and the emission factor of the corresponding fuel as given below:

$$E = \sum \text{Fuel}_j \times \text{ef}_j,$$

(17)

where, E is emission rate of pollutant (g/day), Fuel$_j$ is consumption of fuel j (kg/day), ef$_j$ is emission factor of pollutant per unit consumption of fuel j (g/kg). The emission factors are taken from Gurjar et al. (2004) and the data of cooking gas, kerosene oil, fuel wood, crops waste and dung cake during the period 2008–09 has been taken from Delhi statistical book (2010). The emission of pollutant has been distributed with respect to population density in all the grids. The emission of PM$_{10}$ form industries has been collected during the period 2008–09 has been taken from Delhi Pollution Control Committee (DPCC) website (http://dpcc.delhigovt.nic.in/dpcc_remote/ lis/viewTPPReportsView.php).

**WRF Model Configuration and Initialization**

The WRF-(Advance Research WRF) ARW is a next generation, fully compressible, Euler non-hydrostatic mesoscale forecast model with a run-time hydrostatic option. The model is useful for downsampling of weather and climate ranging from a kilometer to thousand of kilometers and useful for deriving meteorological variables required for air quality models. The WRF model is adopted for simulating the hourly meteorological parameters for 1 Dec to 30 Dec 2008 with 24 h slice interval with two way nesting option in this study. The model uses terrain-following hydrostatic pressure coordinate system with permitted vertical grid stretching (Laprise, 1992). Arakawa-C grid staggering is used for horizontal discretization. The model equations are conservative for scalar variables. The detailed description of WRF is presented in Wang et al. (2004). The computational domains of 70 $\times$ 70 $\times$ 51, 91 $\times$ 91 $\times$ 51 and 55 $\times$ 52 $\times$ 51 grid points and horizontal resolutions of 27, 9 and 3 km, respectively, have been chosen in this study. The model is initialized by real boundary conditions using NCAR-NCEP’s Final Analysis (FNL) data (NCEP-DSSI, 2005) having a resolution of 1$^\circ$ $\times$ 1$^\circ$ (~111 km $\times$ 111 km). A ratio of 1:3 is maintained between resolutions of the outer domain and FNL data to ensure reliable boundary conditions for the model. The microphysical sub grid processes are represented by the scheme described by Lin et al. (1983). The Kain-Fritsch Scheme (Kain and Fritsch, 1993) is used to represent cumulus parameterization. Rapid Radiative Transfer Model (RRTM) long-wave radiation parameterization (Mlawer et al., 1997) and short wave radiation parameterization by Dudhia (1989) are used to represent the long wave and short wave radiation process, respectively. The ACM2 (Pleim) PBL parameterization (Pleim, 2007) is used to represent the PBL over domain. The land surface process is represented by thermal diffusion scheme and surface layer is based on similarity theory (Janjic, 2002).

**RESULTS AND DISCUSSION**

**Validation of Present Analytical Model**

The model presented in above section is evaluated with the tracer observations obtained from Prairie Grass diffusion experiment conducted at O’Neill, Nebraska (Barad, 1958).

**Prairie Grass Experiment**

Prairie Grass experiment is still one of the standard and widely used dataset of dispersion observations utilized for the evaluation of dispersion models for ground level releases of continuous plume over flat terrain. In all 68 test runs of this experiment (Barad, 1958), a tracer sulphur dioxide ($SO_2$) was released without buoyancy at the height of 0.46 m (except four runs# 65–68 in which the release height was 1.5 m) and samples of the concentrations of $SO_2$ were measured at 1.5 m above the ground surface on five sampling arcs 50, 100, 200, 400 and 800 m downwind from the source. The wind speed was measured at seven
vertical levels between 0.25 m and 16 m. The digital version of dataset (http://www.dmu.dk/International/Air/Models/Background/ExcelPrairie.htm) has been used for normalized crosswind integrated concentrations and meteorological variables in this study. In this dataset, the meteorological parameters monin obokavo length (L), frictional velocity (u*), convective velocity scale (w*) and boundary layer height (h) are given and the surface roughness length z0 was reported as 0.006 m. This dataset contains a large number of samples including the low and moderate wind conditions. The low wind (wind speed ≤ 2 m/s) conditions runs (Run# 2,3,4,13,14,32,36,37,40,41,55,60) have been considered for validation the present analytical model. The mean wind speed at the reference height at 4 m level and the release and sampling heights are taken as 0.46 m and 1.5 m respectively. The crosswind integrated concentrations computed from the present model is compared with observed crosswind integrated concentrations for the near source at 50 m and 100 m arc in the form of Q-Q plot as Fig. 1 and the present model predicts 79.16% cases in a factor of two (FAC2) to observations. The correlation coefficient (R), root mean square error (RMSE) and normalized mean square error (NMSE) between the computed and observed concentrations are 0.79, 1192.66 and 0.0026 respectively. The value of correlation coefficient shows that the present model has a good correlation with the observations. Fig. 1 also compares the normalized crosswind integrated concentrations computed from the analytical model including the downwind diffusion (present), AERMOD (Cimorelli et al., 2005) and analytical model without considering the downwind eddy diffusivity (without Kx). AERMOD is a steady state and Gaussian-based plume dispersion model with significant improvements over commonly applied regulatory dispersion models. It includes the effects on dispersion from vertical variations in the atmospheric boundary layer. The crosswind integrated concentrations by AERMOD model for each run are computed by Olesen et al. (2007) and are available from him as well as on NERI’s website (http://www.dmu.dk/International/Air/Models/Background/ExcelPrairie.htm). The statistical evaluation of all three models computed concentration has been shown in Table 1, where C_o and C_p are the observed and predicted concentrations, respectively, while σ is the standard deviation. Table 1 shows the value of present model has the minimum RMSE than the AERMOD and analytical model without considering the downwind eddy diffusivity. On the other hand, 87.5% cases are predicted with in a FAC2 by AERMOD in comparison to 79.16% and 83.33% cases from the present and without considering the eddy diffusivity analytical model. Overall, the present model has comparable results to AERMOD and analytical model without considering the eddy diffusivity. The performance of the present model is performing well in low wind condition but it can be further improved after considering the better parameterization of eddy diffusivities. The parameterization of the exponent β in the profile of eddy diffusivity is taken as β = 1 – p and is based on the Schmidt’s conjugate law. This formulation of β may not be a proper choice as it is theoretically derived only for neutral conditions and may not represent realistically the features of the eddy diffusivity in stable and unstable conditions.

Fig. 1. Represents the ratios of unpaired predicted and observed concentration in Q-Q plot with observed crosswind integrated concentration normalized by the som, urce strength Q for present, AERMOD and without Kx models for Prairie Grass experiment.
The emission load of PM10 from diesel, petrol and CNG as (81%, 13%, 6%) respectively. The total emissions due to all type of sources are discussed below:

The emission of PM10 pollutant from domestic sources has been calculated for cooking gas, kerosene oil, fuel wood, crop waste, dung cake. The emission rate of PM10 is found to be 2.88 tons/day. The load of PM10 from different industries has been found to be 1.52 tons/day by CPCB (2010). The flow rate of different power plants in Delhi from DPCC (http://dpcc.delhigovt.nic.in/dpcc_remo te/vis/viewTPPReportsView.php) reveals that Badarpur thermal power plant (BTPP) has a major role in contributing the pollutants emissions in Delhi, which is 26.27 tons/day of PM10. The other coal based power plant named Rajghat is small in capacity compared to BTPP and contributes less emission of PM10 in comparison to BTPP, which is quantitatively found to be 3.17tons/day for. The other two power plants namely Indraprastha and Praghati are gas based power plants and the total emission of PM10 from all four power plants is 29.45 tons/day.

The emissions of above pollutant from vehicles have been estimated through IVE model and found to be 12.05 tons/day. The IVE model includes geographical and meteorological parameters namely altitude, temperature and humidity, which shows their effect on emissions (Nagpure et al., 2011). The emissions of different fueled vehicles has also been studied, which reflects the percentage of PM10 pollutant contributed by diesel, petrol and CNG as (81%, 13%, 6%) respectively. The percentage of PM10 from diesel vehicles is found to be 30% of PM 10 respectively. The total emissions due to all type of sources, including road dust emission are spatially distributed in each grid over the study area, which are graphically presented in Fig. 2. The road dust emission as estimated by CPCB (2010) has also been considered in Fig. 2. It is noticeable in this figure that PM10 emission ranging from 0.0025 to 26.65 tons/day has high values in grids of power plants and major traffic intersections like I.T.O, A.I.I.M.S, Dhaula Kuan and Panjabi Bagh etc.

**Validation of WRF Model**

The scattered diagram between hourly observed surface dry bulb temperature (DBT) monitored by Indian Meteorological Department (IMD) at Safdarjung Airport and WRF model simulated DBT at 2 m is shown in Fig. 3. The computed and observed surface DBT show a significant similar trend with correlation coefficient (R) about 89% for Dec, 2008. The deviation of the simulated DBT from observed ranges are between −7.12°C (10 UTC of 18 Dec) and +7.86°C (05 UTC of 30 Dec). The other meteorological variables namely dew point temperature, monitored at 2 m; relative humidity, monitored at 2 m and wind speed, monitored at 10 m by IMD are also analyzed in the form of RMSE, NMSE, R and fractional bias (FB) as statistically error analysis. The statistical evaluation of WRF model's forecasted meteorological variables namely DBT, dew point temperature, relative humidity, wind speed with same observed meteorological variables has been made and is given in Table 2, which shows that the RMSE between observed and model simulated DBT, dew point temperature, relative humidity and wind speed is 2.46, 2.93, 19.16 and 1.24 respectively for Dec, 2008. This table also reflects that DBT and wind speed are under-predicting. However, dew point temperature and relative humidity are over-predicting to the observation. The distribution of DBT deviation show that about 31%, 57%, 75% and 95% deviations lie in the ranges of ±1°C, ±2°C, ±3°C and ±4°C, respectively for Dec month. The positive (negative) temperature deviation can affect the stability in analytical model but it is estimated through the temperature differences at different levels. The resulting deviation if any, in the predicted concentrations will be minor and limited to that particular hour in the day. Such minor deviations in hourly concentrations are not likely to significantly affect the 24 hourly averaged concentrations in the final results. Thus it can be concluded that WRF model is able to simulate the meteorological variables reasonably well to simulate the concentration from analytical model over the study area.
and WRF model simulated wind direction at 10 m is shown in the form of wind rose in Figs. 4(a) and 4(b), respectively for Dec, 2008. It is observed that out of 720 observational records of wind directions, 2.01%, 8.65%, 41.65% and 47.69% records show that winds are prevailing from northeast, southeast, southwest and northwest directions,

Fig. 2. Spatial distribution of PM$_{10}$ emission from all types of sources over Delhi city.

Fig. 3. Comparison of observed and model simulated surface temperature (°C) at 2 m during 01 Dec to 30 Dec 2008.
Table 2. Statistical evaluation of WRF model's simulated meteorological variables for validation.

<table>
<thead>
<tr>
<th>Statistical Error</th>
<th>DBT</th>
<th>Dew Point Temperature</th>
<th>Relative Humidity</th>
<th>Wind Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>2.46</td>
<td>2.93</td>
<td>19.16</td>
<td>1.24</td>
</tr>
<tr>
<td>NMSE</td>
<td>0.0065</td>
<td>0.0250</td>
<td>0.0041</td>
<td>0.4826</td>
</tr>
<tr>
<td>R</td>
<td>0.8911</td>
<td>0.4955</td>
<td>0.7313</td>
<td>0.5328</td>
</tr>
<tr>
<td>FB</td>
<td>0.0244</td>
<td>-0.0586</td>
<td>-0.1526</td>
<td>0.1693</td>
</tr>
</tbody>
</table>

Fig. 4. Wind rose plots (a) observed (b) model simulated at 10 m during 01Dec to 30 Dec 2008.

respectively. Similarly, simulated wind directions, 2.29%, 5.35%, 12.78% and 79.58% are in northeast, southeast, southwest and northwest directions, respectively. The observed and simulated outputs show maximum percentage in northwesterly direction and minimum percentage in northeasterly direction. The wind direction between observation and WRF are shifted by 22.5 degrees. It can be shifted because of different frequency of monitored wind direction from IMD and simulated wind direction from WRF model. The reason for such a discrepancy could be attributed to the poor initial and boundary conditions as the model is initialized by real boundary conditions using NCAR-NCEP’s Final Analysis (FNL) data (NCEP-DSSI, 2005) having a resolution of $1^\circ \times 1^\circ$ ($\sim 111$ km × 111 km). In case of the wind speeds, the model seems to under predicting the speeds (average speed 1.57 m/s) as compared to those observed (average 1.86 m/s). In addition to this, model is able to capture the calm wind (wind speed $< 1$ m/s). The model is showing around 24% times the calm wind condition in comparison to 31% in observation in Dec month. In addition to this, model is able to capture the calm wind (wind speed $< 1$ m/s). The model is showing around 24% times the calm wind condition in comparison to 31% in observation in Dec month.

Overall it can be concluded that the winds are represented reasonably well by WRF to simulate the concentration from analytical model.

Simulated Dispersion of PM$_{10}$ over Delhi City

A case study of Delhi has been made to test the performance of above model to predict the concentration of PM$_{10}$ over the month of December 2008. The most important parameters of the models are emission inventory and meteorological variables, which are prepared according to the model’s format. The hourly meteorological data, simulated by WRF model is used as input file to the models. Atmospheric stability has been estimated using upper level data and classified according to Pasquill's stability classes of A, B, C, D, E and F, which range from extremely unstable to extremely stable as given by Turner (1969). The dispersion parameters have also been calculated based on the stability parameters as discussed in above section.

In the above solutions, the exponent $p$ has relationship with Pasquill’s Stability Classes (Hanna et al., 1982) and $\beta = 1 - p$ is based on the Schmidt’s conjugate law. In the modified form of $K_z(x, z)$, the correction function $f(x)$ is taken from Mooney and Wilson (1993) as: $f(x) = [1 - \exp(-x/L_1)] = [1 - (1 - x/L_1)(x/L_1)^2\ldots] = x/L_1$ (Taking only first approximation), where $L_1$ is the along-wind length scale and depends on vertical turbulent intensity. Similarly, in the modified form of $K_x(x, z)$, the correction function $g(x)$ is defined as: $g(x) = x/L_2$, where $L_2$ depends on downwind turbulent intensity. In the above solutions, standard deviation of concentration distribution in crosswind direction is represented by a power of downwind distance...
σ_y = Rx^r (Seinfeld, 1986). These R and r are constants depending on the atmospheric stability. The plume rise height for power plants is also calculated by using stack diameter, stack gas speed and stack gas temperature (Seinfeld, 1986). The spatial distribution of the concentration of PM_{10} pollutants over Delhi city due to all type of sources as point, line and area sources has been shown in Fig. 5. The concentrations from thermal power plants, vehicular traffic and domestic sources are computed as solution of point (Eq. (13)), line (Eq. (15)) and area (Eq. (16)) sources respectively. However, industrial areas are also considered as area sources in the present study. The simulated maximum concentration of PM_{10} during the period ranges between 300 and 460 µg/m^3. The 24 hourly PM_{10} levels are more near to major traffic intersection, power plants and industrial regions of city as compared to other areas.

Comparison of Observed and Computed Concentration of PM_{10}

Comparison of the analytical model output with observed air quality values at four locations in Delhi city is shown as scatter plot in Fig. 6. The output of analytical model includes the background concentrations i.e., baseline pollutant levels or those transported from sources outside the study area (35 µg/m^3 from Sengupta, 2008). The model predictions are well within a factor of two as 91.66% for Dec 2008, which satisfies the criteria of Chang and Hanna (1982) for assessing the performance of the model. The results show that the concentration levels of the models are always higher than the National Ambient Air Quality Standards (100 µg/m^3). The statistical coefficients as RMSE, NMSE and (index of agreement) IA between observed and model simulated concentrations of PM_{10} are 107.10, 0.1987 and 0.4904 for Dec 2008 respectively, which shows that model is performing satisfactory. However, the same methodology has been used to predict the concentration of PM_{10} in other months. In order to make the presentation in a manageable size, the presentation has been made only for Dec month.

CONCLUSIONS

In the present study, the analytical models for dispersion of air pollutants released from point, line and area sources are formulated by considering the wind speed as a power law profile of vertical height above the ground and horizontal and vertical eddy diffusivity as an explicit function of downwind distance from the source and vertical height in different boundary conditions, which is also applicable to simulate the concentration in low wind. The present model has been compared with AERMOD and without including downwind diffusion analytical models in low wind condition of Prairie Grass experiment. The model with Neumann
boundary condition has been used to predict 24 hourly concentration of PM$_{10}$ in the month of Dec 2008 for Delhi. In this study an emission inventory of PM$_{10}$ emitted from different types of primary and secondary sources namely vehicular, domestic, industries and power plants using IVE model has been made for Delhi. The meteorological variables have been simulated using WRF model. The analytical model is evaluated with observed concentration at different locations in Delhi obtained from CPCB, which show that the model is performing satisfactory and within a factor of two with observation. The results also show that the concentration levels are always high in comparison to the National Ambient Air Quality Standards. Although the present model has the limitation as dry deposition is neglected in the present study.

Thus, the methodology of coupling a prognostic regional model with an air pollution model for simulating pollutant dispersion has shown encouraging results and the system has potential to overcome the limitation of unavailability of required local meteorological observations. Improvement in the emission rate of the pollutant, use of meteorological data assimilation for updating the initial and boundary conditions in WRF model will be required to improve the predictability of pollutant dispersion in the future. The performance of the present model can be further improved after considering the better parameterization of eddy diffusivities. The present analytical models can also be used for forecasting after combining with statistical models.

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APPENDIX A

Based on the analysis of Prairie Grass and some other historical tracer experiments of atmospheric dispersion, Irwin et al. (2007) concluded that the ground level crosswind concentration profile of dispersing plume on average is
well characterized as having a Gaussian shape, which is well predicted by all atmospheric transport and diffusion models, regardless of their sophistication. Thus, by assuming the Gaussian concentration distribution in crosswind direction (Huang, 1979; Irwin et al., 2007), the steady state concentration of a pollutant released from point source in a three dimensional domain can be described as

$$C(x, y, z) = C(x, z) \frac{\exp(-y^2/2\sigma_y^2)}{\sqrt{2\pi}\sigma_y}. \quad (A1)$$

where $C(x, z)$ is the crosswind integrated concentration and can be obtained using the separation of variable technique:

By substituting the wind profile (Eq. (7)) and diffusivity profile (Eq. (8)–Eq. (11)) in the advection-diffusion equation for crosswind integrated concentration, one obtains:

$$\frac{\partial C}{\partial x} = \frac{c}{a} \frac{\partial}{\partial x} \left( g(x) \frac{\partial C}{\partial x} \right) + \frac{b}{a} \frac{\partial}{\partial z} \left( f(x) \frac{\partial}{\partial z} \left( -\frac{\partial C}{\partial z} \right) \right) \quad (A2)$$

provided $g(x)$ and $f(x) \neq 0$, $\forall x \in (0, \infty)$. By using the separation of variables technique, the solution of the Eq. (A2) is assumed in the form:

$$C(x, y) = X(x) Z(z). \quad (A3)$$

This separated form Eq. (A3) of the solution, transforms the Eq. (A2) into two ordinary differential equations:

$$A \frac{dX}{dx} - B \frac{d}{dx} \left[ x \frac{dX}{dx} \right] + \lambda^2 X = 0 \quad (A4)$$

$$\frac{d}{dz} \left( z^2 \frac{dZ}{dz} \right) + \lambda^2 \left( \frac{a}{b} \right) Z = 0 \quad (A5)$$

where $\lambda^2$ is a separation constant.

Eq. (A4), along with $f(x) = x/L_1$ and $g(x) = x/L_2$:

$$A \frac{dX}{dx} - B \frac{d}{dx} \left[ x \frac{dX}{dx} \right] + \lambda^2 x X = 0 \quad (A6)$$

where $A = L_1$ and $B = cL_1/aL_2$.

Let $X(x) = x^r W(x)$ and $r = A/2B$, Eq. (A6) becomes

$$x^2 \frac{d^2 W}{dx^2} + x \frac{dW}{dx} - W \left[ \frac{1}{2} x^2 + \frac{1}{2} \lambda^2 x^2 \right] = 0. \quad (A7)$$

This is a Bessel equation and solution will be

$$W(x) = [C_i \left( \frac{\lambda x}{\sqrt{B}} \right) + C_2 K_i \left( \frac{\lambda x}{\sqrt{B}} \right)] \quad (A8)$$

where $I_i$ and $K_i$ are modified Bessel function of first and second order.

Now $X(x) = x^r \left[ C_i \left( \frac{\lambda x}{\sqrt{B}} \right) + C_2 K_i \left( \frac{\lambda x}{\sqrt{B}} \right) \right] \quad (A9)$

Since $X(x) \rightarrow 0$ as $x \rightarrow \infty$, $C_1 = 0$.

So now $X(x) = x^r \left[ C_2 K_i \left( \frac{\lambda x}{\sqrt{B}} \right) \right] \quad (A10)$

Eq. (A5), along with the following boundary conditions corresponding to Eqs. (2) and (3):

$$-b \frac{dz}{dz} = 0 \quad (A11)$$

represents a Sturm-Liouville eigen value problem.

For $\lambda = 0$, the solution of Eq. (A5) with the boundary conditions (A11) is an arbitrary constant $A_0$. For a non-zero value of $\lambda$, the transformation of the variables

$$\beta = \sqrt{b/a} \lambda \quad (A12)$$

and

$$Z(z) = z^{1/2} G(t) \quad (A13)$$

in Eq. (A5) leads to:

$$t^2 \frac{d^2 G}{dt^2} + t \frac{dG}{dt} + \left( k^2 t^2 - \mu^2 \right) G = 0, \quad (A14)$$

where

$$k = \sqrt{a/b} \lambda \quad (A15)$$

Eq. (A14) is the Bessel’s equation and its solution is given by:

$$G(t) = B_1 J_\mu(kt) + B_2 J_{-\mu}(kt), \quad (A16)$$

where $J_\mu$ and $J_{-\mu}$ are the Bessel’s functions of first kind of order $\mu$ and $-\mu$ respectively. From Eqs. (A12), (A13) and (A16), we have

$$Z(z) = z^{1/2} \left[ B_1 J_\mu \left( k z^{1/2} \right) + B_2 J_{-\mu} \left( k z^{1/2} \right) \right]. \quad (A17)$$

Application of the boundary condition (A11) at $z = 0$ in Eq. (A17) yields $B_1 = 0$ and the condition at $z = h$ (Eq. (A7)) gives rise:

$$J_{-\mu}(k h^{1/2}) = 0. \quad (A18)$$
The corresponding eigen functions are:
\[ Z_n(z) = z^{-\frac{1}{2}} J_{\mu} \left( k_n z^{(p-\beta+2)/2} \right), \quad n = 1, 2, 3, ... \] (A19)

Using the Eqs. (A5), (A14) and (A19), the general solution of Eq. (A2) is given by
\[ C(x, z) = A_0 + z^{-\frac{1}{2}} \sum_{n=1}^{\infty} D_n J_{\mu} \left( k_n z^{(p-\beta+2)/2} \right) x^\mu K_1 \left( \frac{\lambda_n x}{\sqrt{B}} \right) \] (A20)

where \( A_0, D_1, D_2, \ldots \) are the unknown coefficients. The Eq. (A20) gives the representation of the concentration distribution \( C \) as the Fourier-Bessel series (Abramowitz and Stegun, 1972) corresponding to a set of eigen functions \( Z_n \).

**Estimation of the coefficients \( A_n \)’s:**

The source condition at \( x = 0 \) (Eq. (6)), gives
\[
Q_x = \frac{Q_p (p+1)}{a h^{p+1}}. 
\] (A22)

Again, multiplying Eq. (A21) by \( z^{(p+1)/2} J_{\mu} (k_n z^{(p-\beta+2)/2}) \), \( m \geq 1 \), and integrating it with respect to \( z \) from 0 to \( h \), we get:
\[
D_n = \frac{Q_p (p+1) \lambda_n}{a \Gamma 2^{-1} (\sqrt{B})^h h^{p+2} \mu} \frac{z^{\frac{1}{2}} J_{\mu} \left( k_n z^{(p-\beta+2)/2} \right)}{J_\mu^2 \left( k_n h^{(p-\beta+2)/2} \right)} .
\] (A23)

Substituting the expression for \( A_0 \) and \( D_n \)’s, \( n \geq 1 \), from Eqs. (A22) and (A23) into Eq. (A20), the final solution is given by
\[
C(x, z) = Q_p \left[ \sum_{n=1}^{\infty} \frac{(\lambda_n)^m J_{\mu} \left( k_n z^{(p-\beta+2)/2} \right) J_\mu \left( k_n (z)^{p-\beta+2} \right)}{J_\mu^2 \left( k_n h^{(p-\beta+2)/2} \right)} \times K_1 \left( \frac{\lambda_n x}{\sqrt{B}} \right) \right] 
\] (A24)

in which
\[
\gamma_n = k_n h^{\frac{p-\beta+2}{2}} .
\] (A25)

After substituting the Eq. (A24) in to Eq. (A1) to get the concentration in to 3D
\[
C(x, y, z) = Q_p \left[ \sum_{n=1}^{\infty} \frac{(\lambda_n)^m J_{\mu} \left( k_n z^{(p-\beta+2)/2} \right) J_\mu \left( k_n (z)^{p-\beta+2} \right)}{J_\mu^2 \left( k_n h^{(p-\beta+2)/2} \right)} \times K_1 \left( \frac{\lambda_n x}{\sqrt{B}} \right) \right] 
\] (A26)

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